ON THE THEORY OF AXISYMMETRIC TURBULENCE

(K TEORII OSESIMMETRICHNOI TURBULENTNOSTI)

PMM Vol.29, № 2, 1965, p.356

M.N.REPNIKOV

(Moscow)

(Received October 28, 1964)

Established turbulent flow of an incompressible fluid across a plane infinite lattice is axisymmetric.

Let the z-axis of a Cartesian coordinate system be normal to the plane of the lattice, and denote by u, v, w and u', v', w' the velocity components at the two points $x = 0$, $y = 0$, z and $x' = 0$, $y' = r$, z' respectively. Then the time averages of the products of components are functions of the three independent variables r, z, z'

If the averages of the triple products are neglected in comparison with the double products, then it follows from the Navier-Stokes and continuity equations, in particular,

$$\frac{W}{\nu}\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial z'}\right]\langle ww'\rangle = \left[2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial z'^2}\right]\langle ww'\rangle$$

where

$$\langle ww'\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T ww'\,dt, \qquad W = \langle w\rangle = \langle w'\rangle$$

A solution of this equation (of source type) has the form

$$\langle ww'\rangle = \operatorname{Re}\left\{\frac{\zeta+\zeta'+i(\zeta-\zeta')}{\zeta^2+\zeta'^2}\exp\left[-\frac{\eta^2}{8}\frac{\zeta+\zeta'+i(\zeta-\zeta')}{\zeta^2+\zeta'^2}\right]\right\}$$

Here

$$\eta = \frac{W}{\nu}r, \qquad \zeta = \frac{W}{\nu}z, \qquad \zeta' = \frac{W}{\nu}z'$$

We have

$$\langle w^2\rangle = \frac{1}{\zeta} \qquad \text{for } \eta = 0, \quad \zeta = \zeta'$$

Thus in this case anisotropy leads to decay of the turbulent energy according to the law z^{-1}.

The author thanks M.D. Millionshchikov for help and discussion.

Translated by M.D.V.D.